Problems from the American Mathematical Monthly, due May 31, 2026

12576. *Proposed by Erik Vigren, Swedish Institute of Space Physics, Uppsala, Sweden.* For positive integers k and n greater than 2, let $f_k(n)$ be the largest positive integer m such that $m! < (n!)^k$. For example, $f_3(5) = 9$ because $9! < (5!)^3$ but $10! > (5!)^3$.
(a) Prove that if $k \in \{2, 3\}$, then $f_k(n+1) - f_k(n) \in \{k-1, k\}$ for all n.
(b) Prove that if $k \geq 4$, then $f_k(n+1) - f_k(n) \in \{k-1, k\}$ for sufficiently large n.

12577. *Proposed by Haoran Chen, Suzhou, China.* When A, B, and C are points in the plane with $\angle BAC < \pi$, we say that $\angle BAC$ is *well-divided* by a finite set of interior points if, when the interior points are $X_1, \ldots, X_k$ in angular order from B to C, we have $\angle BAX_1 = \angle X_1AX_2 = \cdots = \angle X_kAC$. Suppose that we have n points in the plane, with $n \geq 5$ and with no three of the points collinear, and suppose that any angle formed by three of the points is well divided by the other points (if any) that lie inside the angle. Prove that the points form a regular n-gon.

12578. *Proposed by Navid Safaei, Bulgarian Academy of Sciences, Sofia, Bulgaria.* Find all functions $f : \mathbb{R}^+ \to \mathbb{R}^+$ such that $f(x + 4f(y)) = f(x + 3y) + f(y)$ for all positive real numbers x and y.

12579. *Proposed by Andrei Vila, International Computer High School of Bucharest, Romania, and Robert Rogozsan, Baia Mare, Romania.* Let A and B be n-by-n complex matrices with $n \geq 3$ such that $AB = BA^{n-1} + I$, where I is the n-by-n identity matrix. Prove that A is invertible.

12573. *Proposed by Dan Dima, Bucharest, Romania, and Peter Winkler, Dartmouth College, Hanover, NH.* A certain bar offers n seats in a line. The first customer to enter can sit anywhere. Subsequent patrons arrive one at a time and are social-distancing; they will leave if all seats are occupied or adjacent to an occupied seat, and otherwise will choose a seat such that the distance from their seat to their nearest neighbor is as large as possible. For which n can the first customer choose a seat that will result in maximal occupancy, in other words, that will result in $\lceil n/2 \rceil$ seats being occupied?

12574. *Proposed by Tuan Anh Nguyen, Nguyen Quang Dieu High School for the Gifted, Dong Thap, Vietnam.* Let $L_1 = 1$, $L_2 = 3$, and $L_m = L_{m-1} + L_{m-2}$ when $m \geq 3$. (These are the *Lucas* numbers.) A *composition* of n is a list of positive integers whose sum is n. For example, the compositions of 4 are (4), (3,1), (1,3), (2,2), (2,1,1), (1,2,1), (1,1,2), and (1,1,1,1). Given a composition c, let $o(c)$ and $e(c)$ be the number of odd parts and even parts, respectively, of c. Prove

$$\sum 3^{o(c)}(-2)^{e(c)} = L_{2n},$$

where the sum is taken over all compositions c of n.

12575. *Proposed by Hüseyin Yiğit Emekçi, Izmir, Turkey, and George T. Gilbert, Texas Christian University, Fort Worth, TX.* Let r be a real number with $r \geq 2$, and let $b_1, \ldots, b_n$ be positive real numbers satisfying $\sum_{i=1}^{n} b_i = n$. Prove

$$\sum_{i=1}^{n} \frac{1}{b_i^r - b_i + n} \leq 1,$$

with equality if and only if $b_i = 1$ for all i.

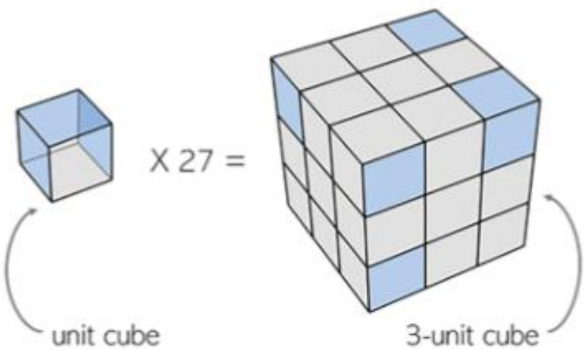
Problems from Math Horizons, due May 31, 2026

Sum of Sines (P494). Ángel Plaza (Universidad de Las Palmas De Gran Canaria, Spain) proposed this problem. Evaluate

$$\lim_{n\to\infty}\sum_{k=1}^{n}\sin\left(\frac{n}{n^2+k^2}\right).$$

Random Cubes (P495). Arsalan Wares (Valdosta State University) suggested this problem. Twenty-seven identical unit cubes are to be randomly glued together to form a three-unit cube, as shown in figure 1. Each unit cube has three blue and three gray faces. Two of the blue faces are opposite each other and the third blue face is opposite a gray face. Determine the probability that 4/27 of the total surface area of the resulting three-unit cube is blue.

Figure 1. A blue and gray unit cube and three-unit cube.



Searching for 4-4 Soaisu (P496). Kenichi Takemura (Japan) challenges readers with this problem connected to his article "The Magic of a 12-12-12 Split: Discovering Soaisu." Fill a 4×4 grid with the numbers 1 through 16 in the natural way. Define a 4-4 soaisu of level 3 to be a coloring of the grid with four blue numbers and four red numbers with the property that the sums of the first, second, and third powers of the four blue numbers equal those of the four red numbers. For example, figure 2 shows a 4-4 soaisu as $2^n+8^n+9^n+15^n=3^n+5^n+12^n+14^n$ for $n=1,2,3$.

Figure 2. A 4-4 soaisu in a 4×4 grid.

Are there any other 4-4 soaisu of level 3 hidden in the same grid?

Three Squares and a Cube (P497). Danesh Forouhari (San Francisco Bay Area) posed this problem. Let a, b, c, and d be integers such that a and c are even, b is odd, and $0<a<b<c$. Find the smallest values of a, b, c, and d satisfying $a^2+b^2+c^2=d^3$.

Problems from KME Problem Corner, due May 31, 2026

Problem 937. *Proposed by José Luis Díaz-Barrero, Barcelona Mathematical Circle, Barcelona, Spain.*

Three roots of the equation $x^4 - px^3 + qx^2 - rx + s = 0$ are tan A, tan B, tan C where A, B, C are the angles of a triangle ABC. Determine the fourth root as a function of only p, q, r and s.

Problem 938. . *Proposed by José Luis Díaz-Barrero, Barcelona Mathematical Circle, Barcelona, Spain.*

If a,b,c are positive reals no larger than one, prove that

$$\frac{2a - \sqrt[3]{abc}}{1+a} + \frac{2b - \sqrt[3]{abc}}{1+b} + \frac{2b - \sqrt[3]{abc}}{1+b} \geq \frac{3\sqrt[3]{abc}}{1+\sqrt[3]{abc}}$$

Problem 939. *Proposed by Toyesh Prakash Sharma and Etisha Sharma, Agra College, Agra, India.*

Find the highest power of 5 which is contained in 777!

Problem 940. *Proposed by John Zerger, Catawba College, Salisbury, NC.*

Show that if p and q are two consecutive odd prime numbers then $p + q$ is the product of at least three prime numbers (not necessarily distinct),

Problem 941. *Proposed by Guillermo Garcia (student) and Angel Plaza, Universidad de Las Palmas de Gran Canaria, Spain.*

Evaluate the sum $\int artanx\left(e^x + \frac{1}{e^x}\right)dx + \int \frac{1}{1+x^2}\left(e^x - \frac{1}{e^x}\right)dx.$

Problem 942. *Proposed D.M. Bătineţu-Giurgiu, "Matei Basarab" National College, Bucharest, Romania and Neculai Stanciu, "George Emil Palade" School, Buzău, Romania.*

If ((a_n) with n≥1 is a positive real sequence such that $\lim_{n\to\infty} \frac{a_{n+1}}{na_n\sqrt[n]{n!}} = a$ which is a positive real, then compute $\lim_{n\to\infty} \frac{1}{n}\left(\sqrt[n+1]{a_{n+1}} - \sqrt[n]{a_n}\right)$.

Problem 943. *Proposed D.M. Bătineţu-Giurgiu, "Matei Basarab" National College, Bucharest, Romania and Neculai Stanciu, "George Emil Palade" School, Buzău, Romania.*

If ((a_n) with n≥1 is a positive real sequence such that $\lim_{n\to\infty} \frac{a_{n+1}}{a_n\sqrt[n]{n!}} = \pi$, then compute $\lim_{n\to\infty} \left(\sqrt[n+1]{a_{n+1}} - \sqrt[n]{a_n}\right)$.

Problem 944. *Proposed by the editor*.

Find a positive integer x which is divisible by a fourth power and x+1 is divisible by a cube, and x+2 is divisible by a square.

Problems from Crux MathemAttic, due June 15, 2026

MA366. *Proposed by Neculai Stanciu.*

Determine all five-digit numbers that, when divided by 4, result in their reversal.

MA367. Suppose the perimeter of a right triangle is p and the hypotenuse has length h.

a) Find the area of the triangle in terms of p and h.

b) Find the smallest and largest values of p (in terms of h) such that there actually is a right triangle with the given perimeter and hypotenuse.

MA368. A box contains an even number n of balls numbered $1, 2, 3, \ldots, n$. If three balls are randomly taken out of the box, without replacement, what is the probability that the number on one of the balls will be the average of the other two?

MA369. An arithmetic series and a geometric series have r as the common difference and the common ratio, respectively. The first term of the arithmetic series is 1 and the first term of the geometric series is 2. If the fourth term of the arithmetic series is equal to the sum of the third and fourth terms of the geometric series, find the three possible values of r.

MA370. Show that if a prime number is divided by 30, the remainder is either one or a prime number.

Problems from Crux Olympiad Corner, due June 15, 2026

OC776. Given a $n \times n$ table with non-negative real entries such that the sums of entries in each column and row are equal, a player plays the following game: The step of the game consists of choosing n cells no two of which share a column or a row, and subtracting the same number from each of the entries of the n cells, provided that the resulting table has all non-negative entries. Prove that the player can change all entries to zeros.

OC777. In the complex plane, consider a square having the following property: the complex numbers to which its vertices correspond are exactly the roots of an equation with integer coefficients $x^4 + px^3 + qx^2 + rx + s = 0$. Find the minimum area of such a square.

OC778. Given a trapezoid $ABCD$ with $AD \parallel BC$, E is a moving point on the side AB. Let O_1, O_2 be the circumcenters of triangles AED and BEC, respectively. Prove that the length of O_1O_2 is a constant value.

OC779. Find all positive integers n such that $n^4 + 4n^3 + 22n^2 + 36n + 18$ is a perfect square.

OC780. Consider a convex 2025-gon. We draw a subset of its diagonals such that each drawn diagonal (excluding the two endpoints) intersects exactly one of the other drawn diagonals. What is the maximum possible number of such diagonals that can be drawn?

Problems from Crux Mathematicorum, due June 15, 2026

5131. *Proposed by Tigran Hakobyan.*

Describe all the triplets (a, b, c) of positive integers greater than 1 having the following property: for any positive integers k and m there exists a positive integer l such that

$$\gcd(a^k - 1, b^m - 1) = c^l - 1$$

5132. *Proposed by Ovidiu Furdui and Alina Sîntămărian.*

Calculate

$$\lim_{n\to\infty} \sqrt[n]{\int_0^1 \left(x + \frac{x^2}{2^2} + \cdots + \frac{x^n}{n^2}\right)^n \mathrm{d}x}.$$

5133. *Proposed by Ziji Hu.*

Let S be a set of points on a plane with the following properties:

(i) S has at least 3 points;

(ii) No three points in S are collinear;

(iii) If A, B, and C are three distinct points in S, then the circumcenter of $\triangle ABC$ is in S.

Find all such finite sets S or prove that such a set does not exist.

5134. *Proposed by Vasile Cîrtoaje.*

For $n \geq 3$, let $a_1, a_2, \ldots, a_n$ be nonnegative real numbers such that $a_1 \geq a_2 \geq \cdots \geq a_n$ and $a_1a_2 + a_2a_3 + \cdots + a_na_1 = n$. Prove that

$$\frac{1}{a_1 + 2} + \frac{1}{a_2 + 2} + \cdots + \frac{1}{a_n + 2} \geq \frac{n}{3}.$$

5135. *Proposed by Tatsunori Irie.*

A transparent cubic box with a side length of 20 contains 1050 fireflies, which we consider as geometric points. Prove that if one covers a part of the interior of this box with a hemispherical dome of radius 2, there exists a position for the dome such that it contains at least 3 fireflies. (Note: Fireflies located on the boundary of the dome are considered to be inside.)

5136. *Proposed by Marius Stănean.*

Let $a \geq b \geq c \geq 0$ be real numbers such that $a+b+c = ab+bc+ca > 0$. Prove that
$$a^2+b^2+c^2+5abc \geq 8+(a-b)^2.$$

5137. *Proposed by Pelegrí Viader.*

Given $k \in \mathbb{N}$, find the solution of the difference equation
$$x_{n+k} + nx_{n+1} - nx_n = 0 \quad \text{for } n = 1, 2, \ldots, \qquad (1)$$
such that $x_n \to 0$ and $nx_n \to 1$.

5138. *Proposed by Nazar Kirgizbaev.*

Let x, y, z be positive real numbers such that $x^2+y^2+z^2 = 3$. Prove that
$$\sum_{cyc} \left(\frac{2xy}{xy+z}\right)^2 \geq 1 + \frac{2}{3}(xy+yz+zx.)$$

5139. *Proposed by Giuseppe Fera, modified by the Editorial Board.*

Find infinitely many cubes in three-dimensional space so that

(i) all vertices have integer coordinates,

(ii) no edge is parallel to a coordinate axis, and

(iii) no cube is an integer multiple of another.

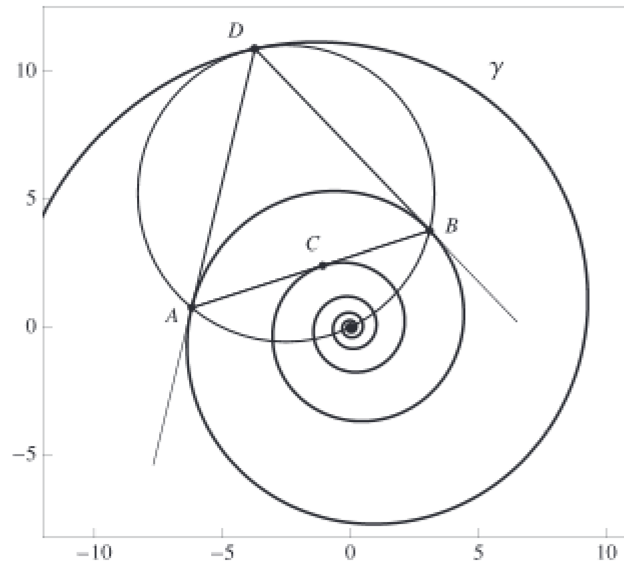
5140. *Proposed by Phan Ngoc Chau.*

Prove that the following inequality
$$\frac{1}{\left(\sqrt{a}+\sqrt{bc}\right)^2} + \frac{1}{\left(\sqrt{b}+\sqrt{ca}\right)^2} + \frac{1}{\left(\sqrt{c}+\sqrt{ab}\right)^2} \geq \frac{2}{abc+1}$$
holds for all positive real numbers a, b, c with $ab+bc+ca = 1$. When does equality occur?

12580. *Proposed by Tran Quang Hung, Hanoi, Vietnam.* For $a > 1$, let γ be the logarithmic spiral with polar equation $r = a^\theta$. Given a point C on γ, let A and B be the two points on γ such that the segment AB is tangent to γ at C but touches γ only at A, B, and C.

(a) Prove that the two tangents to γ at A and B intersect at a point D that is also on γ.

(b) Prove that the circumcircle of $\triangle ABD$ is tangent to γ at D and passes through the origin. (See the figure, where $a = 9/8$.)



12581. *Proposed by Max A. Alekseyev, George Washington University, Washington, DC.* Let ϕ be the Euler totient function. Find all positive integers n such that $\phi(n)$ equals the sum of the prime factors of n taken with multiplicity. That is, find all positive integers n such that $\phi(n) = \sum_{i=1}^{t} k_i p_i$, where n has prime factorization $\prod_{i=1}^{t} p_i^{k_i}$.

12582. *Proposed by Haoran Chen, Suzhou, China, Ilya Bogdanov, Moscow, Russia, and Fedor Petrov, St. Petersburg, Russia.* A train with a fixed number of passenger cars runs along a line with several stations. All passengers have advance reservations specifying the station where they board the train and the station where they leave the train. Prove that, for all possible passenger itineraries, the railroad can assign each passenger to a car so that whenever the train is moving, the numbers of passengers in any two cars differ by at most 1.

12583. *Proposed by Andrés Quintero, student, University of the Andes, Bogota, Colombia, and Stan Wagon, Macalester College, St. Paul, MN.* Let b and q be positive integers with $q \geq 2$ and $\gcd(q, b-1) = 1$. Show that there are three consecutive positive integers each of whose squares has base-b digit sum that is a multiple of q.

12584. *Proposed by Vasile Cîrtoaje, Petroleum-Gas University of Ploieşti, Ploieşti, Romania.* Let $a_1, \ldots, a_n$ be positive real numbers, with $n \geq 4$, and suppose $\sum a_i = n \prod a_i$. Prove

$$\sum_{i=1}^{n} \frac{1}{(n-1)a_i + 1} \geq 1.$$

12585. *Proposed by Aryan Desai, Ahmedabad, India.* Let z be a complex number that is not a nonpositive integer, let Γ be the gamma function, defined by $\Gamma(z) = \int_0^\infty t^{z-1} e^{-t}\, dt$, and let ψ be the digamma function, defined by $\psi(z) = \Gamma'(z)/\Gamma(z)$. Prove

$$\sum_{n=1}^{\infty} \frac{\psi(n+z) - \psi(n)}{n+z-1} = \frac{\pi^2}{6}.$$

12586. *Proposed by Ovidiu Furdui and Alina Sîntămărian, Technical University of Cluj-Napoca, Cluj-Napoca, Romania.* Evaluate

$$\lim_{n\to\infty} n^2 \int_0^\infty \sqrt[n]{\cosh(x) - 1 - \frac{x^2}{2!} - \frac{x^4}{4!} - \cdots - \frac{x^{2n}}{(2n)!}}\, e^{-x}\, dx.$$

Problems from Mathematics Magazine, due July 1, 2026

2236. *Proposed by Cherng-tiao Perng, Norfolk State University, Norfolk, VA.*

Let A be a real number such that $|A| < 1$ and let α and β be complex numbers with $|\alpha| = |\beta| = 1$. Show that if z satisfies

$$z^2 + A(\alpha + \beta)z + \alpha\beta = 0,$$

then $|z| = 1$.

2237. *Proposed by Peng Xicheng, Central China Normal University, Wuhan, China.*

Let $\triangle ABC$ be a triangle with $AB \neq AC$. Let D be the midpoint of BC, H the foot of the perpendicular from A to BC, and L the point where the angle bisector of $\angle BAC$ meets BC. If $m\angle DAL = \alpha$, $m\angle LAH = \beta$, and $m\angle BAC = \gamma$, prove that

$$\tan\alpha \cot\beta = \tan^2 \frac{\gamma}{2}.$$

2238. *Proposed by Peter R. Mercer, Buffalo State University, Buffalo, NY.*

Show that

$$\sum_{n=1}^{\infty} \int_0^1 \frac{dx}{n(1+x^2)^n}$$

converges and find its sum.

2239. *Proposed by Warut Suksompong, National University of Singapore, Singapore.*

Let $P(x)$ be a nonconstant polynomial with integer coefficients and positive leading coefficient. Prove that for every positive integer m, there exists a positive integer k such that $P(k)$ is positive and the decimal representation of $P(k)$ contains at least m 9's.

2240. *Proposed by Anthony J. Bevelacqua, University of North Dakota, Grand Forks, ND.*

When F is a field of characteristic $p > 0$, the map $\wp : F \to F$ given by $\wp(x) = x^p - x$ is an additive homomorphism. Determine all finite fields F such $\wp(F)$ is closed under multiplication.

Problems from the College Mathematics Journal, due July 15, 2026

1316. *Proposed by Dixon J. Jones, Coralville, IA.*

Given a real number $p > 1$ and real constants $a, b \geq 0$, let

$$f_{a,b}(x) = a + (b + x^{-p})^{-p}$$

for $x > 0$. Show that there exists a constant $c > 0$ such that the curves $y = f_{c,0}(x)$ and $y = f_{0,c}(x)$ are tangent to the line $y = x$.

1317. *Proposed by Ángel Plaza, Universidad de Las Palmas de Gran Canaria, Spain.*

Calculate the following sum, with proof:

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{n} \left(\frac{1}{2n-1} - \frac{1}{2n} + \frac{1}{2n+1} - \cdots \right).$$

1318. *Proposed by Paul Bracken, University of Texas Edinburgh, TX.*

Evaluate the following integral, with proof:

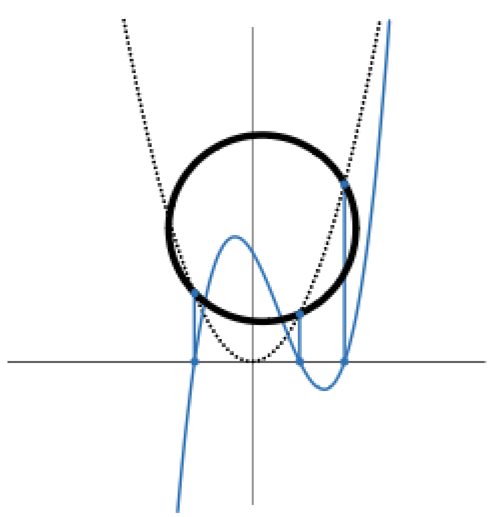
$$\int_0^1 \frac{\tan^{-1}(\sqrt{1-t^2})}{1 \pm t}\, dt.$$

1319. *Proposed by Gregory Dresden, Washington & Lee University, VA.*

Consider the cubic

$$y = x^3 + Ax^2 + Bx + C$$

with three distinct real roots. For each root, extend a line vertically up to the parabola $y = x^2$, giving three points of intersection. Find the expanded equation of the circle through those three intersection points, in terms of A, B, and C.



1320. *Proposed by Vasile Cîrtoaje, Petroleum-Gas University of Ploiesti, Romania.*

Prove that 4 is the largest positive value of k such that the inequality

$$\frac{1}{a^3}+\frac{1}{b^3}+\frac{1}{c^3} \geq a^k+b^k+c^k$$

holds for any positive real numbers a, b, c with at most one of them less than 1 and $a+b+c=3$.

Problems from the American Mathematical Monthly, due July 31, 2026

12587. *Proposed by Roberto Tauraso, Tor Vergata University of Rome, Rome, Italy.* A partition of a positive integer n may be viewed as a strictly increasing list of positive integers $k_1, \ldots, k_r$ and a list of positive integers $m_1, \ldots, m_r$ with $m_1 k_1 + \cdots + m_r k_r = n$. Let $f(n) = \sum (-1)^{m_1+\cdots+m_r-r} m_1 \cdots m_r$, where the sum is over all partitions of n. For example, if $n = 4$, then the partitions are $4 \cdot 1$, $2 \cdot 1 + 1 \cdot 2$, $2 \cdot 2$, $1 \cdot 1 + 1 \cdot 3$, and $1 \cdot 4$, and therefore $f(4) = -4 - 2 \cdot 1 - 2 + 1 \cdot 1 + 1 = -6$. Show that if n is not the sum of two squares, then 5 divides $f(n)$.

12588. *Proposed by H. A. ShahAli, Tehran, Iran.* A *flip* of a $\{0, 1\}$-matrix is the result of choosing a row and flipping all its entries: changing 0 to 1 and 1 to 0. Call a matrix M *alternating* if for all sequences $M_1, M_2, \ldots$ with $M_1 = M$ and with M_{k+1} a flip of M_k for all $k \geq 1$, the matrices in the sequence alternate between having a column of zeros and not having a column of zeros. Determine all pairs of positive integers m, n such that there exists an alternating m-by-n matrix.

12589. *Proposed by Robert Rogojan, Baia Mare, Romania, and George Ţurcaş, Cluj-Napoca, Romania.* Let G be a finite abelian group with at least three elements. For any group H, let $p(H)$ be the product of all elements of H. Let $s(G)$ be the number of distinct elements $p(H)$ as H varies over all subgroups of G distinct from $\{e\}$ and G, and let $n(G)$ equal the number of elements of G of order 2.

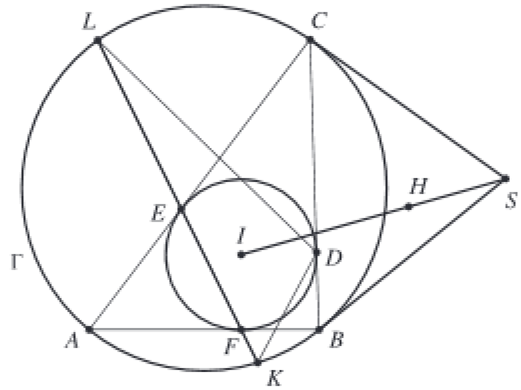
(a) Prove that $n(G) \leq s(G) \leq n(G) + 1$

(b) Find all fiinite abelian groups G such that $s(G) = n(G)$.

12590. *Proposed by Hongwei Lou and Jinjai Yan, Fudan University, Shanghai, China.* Evaluate

$$\lim_{x \to 0^+} x \ln x \sum_{n=2}^{\infty} \frac{\sin(nx)}{\ln n}.$$

12591. *Proposed by Dong Luu, Hanoi National University of Education, Hanoi, Vietnam.* Let $\triangle ABC$ be a triangle with $\angle A \neq 90°$. Let $\triangle ABC$ have circumcircle Γ and incenter I. Let D, E, F be the tangency points of the incircle with BC, CA, AB, respectively. Let K and L be the intersection points of the line through E and F with Γ, and let the tangents to Γ at B and C meet at S. Prove that H, the orthocenter of $\triangle DKL$, lies on the line SI.



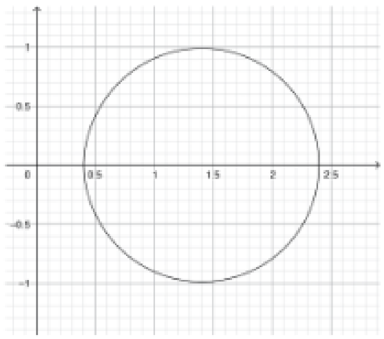
12592. *Proposed by Elliot Glazer, Principia Labs, San Francisco, CA.* Consider a biased coin C with sides labeled 0 and 1. When C is flipped, it shows 0 with unknown probability p and 1 with probability $1 - p$, where $0 < p < 1$. Let the *critical value* of a nonconstant bit sequence be the number of bits in the maximal constant string starting at the beginning.
(**a**) Show how to use C to simulate a fair coin by a method that assigns to any bit sequence with critical value n a result after at most $2n$ flips of C. For example, if the first five flips of C are 00001, then $n = 4$ and the method must declare heads or tails after at most three more flips.
(**b**) Find a simulation method as in part (**a**) that always gives a result after $\max(2, 2n - 1)$ flips of C, where n is the critical value.

12593. *Proposed by Rodney Nillsen, University of Wollongong, Wollongong, Australia.* For a sequence $w_1, w_2, \ldots$ of positive integers, let x be the real number whose binary representation is $0.0^{w_1}1^{w_2}0^{w_3}1^{w_4}\ldots$, where d^{w_n} denotes the string consisting of the digit d repeated w_n times. Show that if $\limsup w_{j+1}/\sum_{k=1}^{j} w_k > 1$, then x is transcendental.

Problems from Math Horizons, due July 31, 2026

Ellipse Investigator (P498). Gregory Dresden (Washington & Lee University) plotted the graph of the polar curve $r = \sqrt{\cos 2\theta} + (7/5)\cos\theta$ in the Cartesian plane for values of θ where $\sqrt{\cos 2\theta}$ exists, shown in figure 1. Circle Sleuth (P441), which appeared in The Playground in September 2022, and its solution in February 2023, showed that this graph is not a circle. Determine (with justification) whether the graph is an ellipse.

Figure 1. A graph of an ellipse?



Inequality with Symmetry (P499). Goran Conar (Varaždin, Croatia) posed this problem. Let $a, b, c > 0$ be real numbers such that $a + b + c = 4$. Prove that $a^a b^b c^c \geq \frac{256}{81}$.

Counting Juggling Balls (P500). Jon Stadler (Capital University) challenges us with a problem connected to his article "Using the Floor to Catch a Dropped Juggling Ball Theorem." An arbitrary sequence $a_0 a_1 \cdots a_{n-1}$ is a juggling sequence provided that the sequence $\sigma = \sigma_0 \sigma_1 \cdots \sigma_{n-1}$ defined by $\sigma_i = i + a_i \pmod{n}$ is a permutation of $0, 1, \ldots, n-1$. Consider the juggling sequences 552534, 420411, 44142, 30011, 4414242, 77272672, 61611414, and 746671464. In each of these juggling sequences $a_0 a_1 \cdots a_{n-1}$, every throw a_i is less than the period of the given sequence. Gather information from these juggling sequences. Using the Floor Theorem from the article, prove that if $a_0 a_1 \cdots a_{n-1}$ is a juggling sequence and $a_i < n$ for all i, then the number of balls equals $b = |\{i : i > \sigma_i\}|$.

A Checkerboard Determinant (P501). Mohammad K. Azarian (University of Evansville) proposed this problem. For $n \geq 1$, let F_n and L_n be the Virahanka-Fibonacci and Lucas numbers, respectively, defined by $F_1 = F_2 = L_1 = 1$, $F_{n+2} = F_{n+1} + F_n$ and $L_{n+1} = F_{n+2} + F_n$. Let

$$D(n) = \begin{vmatrix} 0 & 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & F_n & 0 & F_{n+2} & 0 & L_{n+1} \\ F_{n+3} & 0 & F_{n+5} & 0 & L_{n+4} & 0 \\ 0 & F_n^2 & 0 & F_{n+2}^2 & 0 & L_{n+1}^2 \\ F_{n+3}^2 & 0 & F_{n+5}^2 & 0 & L_{n+4}^2 & 0 \end{vmatrix}.$$

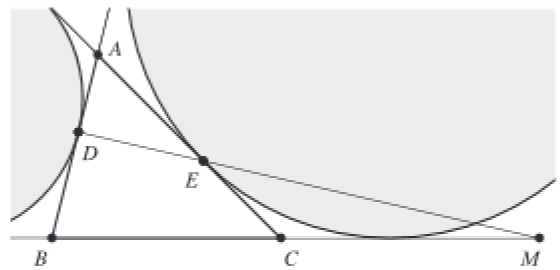
Show that $D(n) = -\prod_{i=0}^{5} F_{n+i}$.

Problems from the American Mathematical Monthly, due August 31, 2026

12594. *Proposed by Hideyuki Ohtsuka, Saitama, Japan.* Let F_n be the nth Fibonacci number: $F_0 = 0$, $F_1 = 1$, and $F_n = F_{n-1} + F_{n-2}$ for all integers n. Prove

$$\sum_{n=-\infty}^{\infty} \arctan\left(\frac{1}{\sqrt{6}F_n + 1}\right) = \sum_{n=-\infty}^{\infty} \arctan\left(\frac{1}{\sqrt{6}F_n - 1}\right) = \frac{\pi}{2}.$$

12595. *Proposed by Yagub Aliyev, ADA University, Baku, Azerbaijan.* Let ABC be a triangle with semiperimeter s and with $\angle C < \angle B < \pi/2$. Let D be the point on side AB where an excircle is tangent, and let E be the point on side AC where an excircle is tangent. Let M be the intersection of lines DE and BC. Prove $DM > s$.



12596. *Proposed by Vahan Mkrtchyan, Purdue University Fort Wayne, IN, and Douglas B. West, University of Illinois, Champaign, IL.* A set of edges in a graph is a *matching* if no two edges in it share a vertex. Among graphs in which the maximum size of a matching is t and no two vertices have the same set of neighbors, determine the maximum number of vertices.

12597. *Proposed by Leonard Giugiuc, Drobeta-Turnu Severin, Romania.* For $n \geq 4$, let $a_1, \ldots, a_n$ be nonzero real numbers such that $1/a_1 + \cdots + 1/a_n = 0$. Prove

$$\left(\frac{1}{a_1^2} + \cdots + \frac{1}{a_n^2}\right) \sum_{1 \leq i < j \leq n} (a_i - a_j)^2 \geq n \left(\frac{a_1}{a_2} + \frac{a_2}{a_3} + \cdots + \frac{a_{n-1}}{a_n} + \frac{a_n}{a_1}\right)^2.$$

12598. *Proposed by Paul Bracken, University of Texas, Edinburg, TX.*

(a) Evaluate

$$\sum_{n=1}^{\infty} (-1)^{n-1} \left(\sum_{k=0}^{\infty} \frac{(-1)^k}{n + 2k}\right).$$

(b) Evaluate

$$\sum_{n=1}^{\infty} (-1)^{n-1} \left(\sum_{k=0}^{\infty} \frac{(-1)^k}{n + 2k}\right)^2.$$

12599. *Proposed by Gerald Bourgeois, Marseille, France.* The *spectral radius* of a square matrix is the maximum of the magnitudes of its eigenvalues. What is the maximum spectral radius among invertible n-by-n $\{0, 1\}$-matrices?

12600. *Proposed by Tho Nguyen Xuan, Hanoi University of Science and Technology, Hanoi, Vietnam.* Let a and b be integers greater than 1, and let $P(x)$ and $Q(x)$ be polynomials with integer coefficients and positive leading coefficient. Suppose $P(x)$ and $Q(x)$ are coprime in $\mathbb{Z}[x]$ and $a^{P(n)} - 1$ divides $b^{Q(n)} - 1$ for all sufficiently large positive integers n. Prove that the degree of P is zero.

Problems from Mathematics Magazine, due September 1, 2026

2241. *Proposed by Tran Quang Hung, Hanoi, Vietnam.*

Given $\triangle ABC$ with altitude AH and median AM (with H and M lying on line BC) such that $AH = MB = MC$. Let E and F be the feet of the perpendiculars from H to CA and AB, respectively. Let P be the midpoint of HM. Prove that line EF bisects segment AP.

2242. *Proposed by Hideyuki Ohtsuka, Saitama, Japan.*

Sylvester's sequence $\{S_n\}$ is defined by $S_0 = 2$ and $S_n = S_{n-1}^2 - S_{n-1} + 1$ for $n \geq 1$. For any integer $n \geq 0$, find closed form expressions for the sums

(a) $\displaystyle\sum_{k=0}^{n} (S_0 S_1 S_2 \cdots S_k)^2$

(b) $\displaystyle\sum_{k=0}^{n} \frac{1}{(S_k S_{k+1} S_{k+2} \cdots S_n)^2}$.

2243. *Proposed by Seán M. Stewart, King Abdullah University of Science and Technology, Thuwal, Saudi Arabia.*

Evaluate

$$\int_0^1 \int_0^1 \frac{\arctan(xy)}{x\sqrt{1-x^2}\sqrt{1-y^2}}\, dy\, dx.$$

2244. *Proposed by Raymond Mortini, Université du Luxembourg, Esch-sur-Alzette, Luxembourg, and Rudolf Rupp, Technische Hochschule Nürnberg, Georg Simon Ohm, Nürnberg, Germany.*

For $k = 1, 2, 3, \ldots$, let $A_k := k(k-1)/2$ and for any real number α, let

$$P_k(\alpha) := \sum_{j=1+A_k}^{A_{k+1}} \frac{1}{j^\alpha}.$$

(a) Evaluate

$$\lim_{k\to\infty} k^{2\alpha} \left(P_{k+1}(\alpha) - P_k(\alpha)\right).$$

(b) Evaluate

$$\lim_{k\to\infty} k^3 \left(P_{k+1}\left(\frac{1}{2}\right) - P_k\left(\frac{1}{2}\right)\right).$$

Note: This problem was inspired by problem 2193 in the April 2024 issue of *Mathematics Magazine*, where the behavior of the series

$$\sum_{k=0}^{\infty} (-1)^k P_k(1/2)$$

was to be investigated.

2245. *Proposed by the Missouri State University Problem Solving Group, Missouri State University, Springfield, MO.*

The propositional formulas

$$\forall x \left(P_1(x) \vee P_2(x) \vee \cdots \vee P_k(x)\right) \text{ and } (\forall x\, P_1(x)) \vee (\forall x\, P_2(x)) \cdots \vee (\forall x\, P_k(x))$$

are not logically equivalent in general. If $|A| = n$ and $P_i : A \to \{T, F\}$, $i = 1, \ldots k$ are logical functions, there are 2^{nk} ordered k-tuples $(P_1, \ldots, P_k)$. For how many of these k-tuples are the propositional formulas above equivalent?